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| Source: | FAU, 450connect (contact: bernhard.gaede;georg.fischer@fau.de) | |
| Subject: | Efficient frequency domain algorithms for calculation of RX Intermodulation | |
| Group membership required to read? (Y/N)  N | | |
|  | | |
| Summary: | | |
| FAU has provided reference values [1] on RX intermodulation inside narrowband PMR receivers in order to support the validation activity of IM-plugin for SEAMCAT currently being conducted in SE7 [3]. The brute force time domain computation of RF signal was compared against a time domain computation in baseband and versus the approximation by 3-frequency-approximation algorithm in frequency domain as presented in [2] (source Motorola). All results are quite in agreement. However FAU found that a faster, non-approximative and more versatile analytical way could be followed, that facilitates consideration of preselectors and BS activity factor in a computational efficient way. The analytical way also avoids the debates in SE7 on spectral widening with approximation by frequency slices.  The analytical way, developed by FAU, is described here. As it is very fast and versatile it has the potential to replace the actual 3-frequency approximation algorithm as coded in the IM-plugin so far. | | |
| Proposal: | | |
| FAU and 450connect invite group to  Replace the actual 3-frequency approximation algorithm in IM-plugin by analytical way presented here  Recognise the versatility and computational speed advantages of analytical way to provide accurate values also in presence of preselector and BS activity factor  Reflect analytical way for calculation of RX intermodulation in the draft ECC report 283  Conduct studies on LTE BS impact on NB PMR MS using a revised IM–plugin that is based on analytical way | | |
| Background: | | |
| SE7 is studying the impact of RX intermodulation in narrowband receivers when exposed to broadband LTE signals in adjacent bands [4]. Outage calculations will be conducted once the IM-plugin is validated. | | |

# Introduction

During the currently ongoing validation activity of IM-plugin [3], FAU compared different ways to compute RX intermodulation. The different ways are listed in the following table:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| ***Algorithm*** | ***Brute force Time domain at RF*** | ***Time domain at equivalent baseband*** | ***3-frequency approximation algorithm in frequency domain*** | ***Analytical way*** |
| Domain | Time | Time | Frequency | Frequency |
| Method | exact | exact | approximative | analytical |
| Suited for SEAMCAT  IM-plugin | no | no | yes | yes |
| Available in SEAMCAT | no | no | Yes (not yet fully validated) | no |
| Computational Effort | Very high | high | moderate | low |
| Verified for use with preselector | yes | yes | ?? | yes |
| Verified with BS activity factor | yes | yes | ?? | yes |

Table 1: Comparison of studied algorithms for IM computation

The Validation activity has revealed that all algorithms deliver in principle consistent results for the case of symmetric LTE signal and 100% activity.

The older 2-frequency approximation algorithm, which still can be selected in current release of IM-plugin, is not listed here as there is consensus in SE7 that it incorrect due to missing convergence.

FAU has delivered reference values based on brute force time domain analysis to support validation activity [1].

# Analytical derivation

The signal (in our case the broadband LTE signal) is sent through a non-linear characteristic in the RF-domain:



In typical use  is selected and  is derived from IIP3:



To find the equivalent expression for the non-linearity in the baseband, we transform a baseband signal



to the RF-domain and plug it into the cubic term:



The inter-modulation products at thrice the carrier frequency can be discarded, as they are irrelevant inside the system:



This is transformed back to the baseband as:



Thus, the baseband representation of the signal  reads:



The LTE-Signal is now modelled as filtered white Gaussian noise, with  being the white noise and  reflecting the impulse response of low pass filter in baseband:



In our approach we select  at the centre of the LTE signal.

For the determination of power spectral density after non-linearity we are interested in the autocorrelation of the output signal after undergoing the nonlinearity:



Each expectation is now carried out individually:





At this point, Isserlis's theorem is used to rewrite the fourth-order moment. For complex Gaussian random variables the following relation holds:



Therefore:



and:



The derivation of the last expectation is completely analogous to the previous ones but very lengthy, hence it is just sketched here:



Again, Isserlis' theorem is used to split the expectation into second order moments yields:



From the previous results we see that terms comprising



yield



and terms comprising



or



and their conjugates result in

.

Obviously for Gaussian noise,



gives

.

Thus:



Collecting the results yields:



The corresponding power spectral density as a function of  (relative to middle of LTE signal) reads:



where

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# Practical Interpretation for computation of spectral regrowth and IM noise

The essential outcome of the analytical derivation is that the spectral regrowth, so the spectral distributions left and right of LTE signal (“spectral shoulders”) are reflected in the 4th term above. The first term is the linear power spectral density followed by 2 terms that reflect IM noise falling inside the LTE signal thus irrelevant for RX intermod in narrowband PMR RX. The 4th term however shows a convolution of the LTE spectrum with itself and its flipped version which contributes to spectral regrowth.

These 2 convolutions in the 4th term look somehow related to the 3 frequency algorithm. One can conclude that the 3-frequency algorithm is an approximation of the analytical way presented here. However, the   
3-frequency algorithm suffers from the missing mathematical justification whether to consider widening by factor 3 (some people propose even 5) or not. And therefore the widening is selectable on the current IM plugin implementation. This problem is avoided with the analytical way shown here.

# Relevance of analtical way for fast computation of IM noise

## General case

As the 4th term is based on convolution of LTE spectrum, it can be replaced by simple multiplication of the corresponding iFFT terms, which i.e. would be a time domain calculation. The product result than needs to be transformed back from time to frequency domain delivering the spectral distribution after nonlinearity.

A complete reverse transformation however is not needed as typically only the spectral bins at the TETRA RX channel are needed to compute the IM noise falling into TETRA RX channel. This can be sped up by the Goertzel algorithm [5, 6].

FFT and iFFT algorithms are known to work extremely fast and optimized code is readily available in the software libraries. On the opposite, nested loops as present in current implementation of IM-plugin are known to slow down computation. FAU & 450connect therefore think a transition from today’s 3-frequency-agorithm to the analytical way could speed up SEAMCAT simulations.

## Specific case

For the case of strict symmetric, flat spectrum of LTE signal - which would be the case for no preselector and 100% activity factor – twofold convolution and integration can be conducted analytically providing an analytically derived closed form expression, which again is no approximation:



with  bandwidth of LTE signal, e.g. 2.7 MHz. The baseband is cantered at the middle of the LTE signal and the TETRA signal is considered to be upper to the LTE signal. Then

 lower edge of TETRA RX channel given in [Hz].

 upper edge of TETRA RX channel given in [Hz].

 Power spectral density of LTE signal given in [W/Hz].

The parameters are illustrated in the following graph:

**TETRA RX**

**LTE**

**f1 f2**

PSD

f

b

gap

*Baseband with analytical way centered at the middle   
of the LTE signal*

Fig. 1: Signal scenario for the ”analytical way” incorporating symbol definitions

## Hybrid scheme

As the analytically derived closed form expression in section 4.2 can be computed very quick, one could also go for a hybrid scheme through first calculating the IM power inside TETRA RX based on the assumption of 100% activity by means of this closed formula and then apply IM reduction due to activity factor of 50% in the range of 6 to 8.5 dB depending on gap as presented in [8].

# Benefits of analytical way for IM noise computation over algorithms looked at so far

In the following a list of arguments is given why a transition from today’s 3-frequency algorithm to the analytical way should be conducted:

## Precision

The 3-frequency algorithm in use so far is an approximation scheme, whereas the analytical way is indeed no approximation. In general in engineering analytical solutions should be given preference over approximations as precision is higher on analytical way than on approximation.

## Spectral spread, power thinning

With the current 3-frequency-algorithm a debate has come up in SE7 whether or not to consider widening of spectrum. A solid mathematical justification whether or not to consider widening is missing. This is unsatisfactory. This debate does not arise on the analytical way. It just delivers the power inside the NB PMR RX channel.

## Speed

The current 3-frequency-algorithm is based on nested loops, which are known to slow down computation. The analytical way however uses FAST FOURIER TRANSFORM and Goertzel Algorithm for fast computation. The analytical way also can make use of optimized code in software libraries.

## Activity factor

ITU has recommended to consider BS activity factor of 50% in coexistance studies [7]. In [8] FAU and 450connect have revealed that BS activity of 50% leads to a reduction of IM noise by 6 to 8.5 dB depending on spectral gap. The results on IM reduction were obtained using the analytical way and verified by brute force time domain calculations at RF.

## Preselector

FAU and 450connect have studied the impact by preselector in NB PMR RX based on analytical way as it is well capable to handle asymmetric spectra, which show up after an LTE signal has underwent a preselector filter. The analytical way provides reliable results as a crosscheck with brute force time domain calculations at RF has revealed. As the solution is purely analytical, there is no approximation in it.

# Summary

FAU has developed a way to compute RX intermodulation through a versatile, fast, analytical and thus precise way. Crosscheck of the analytical way with brute force and thus computational expensive time domain calculation have confirmed the high accuracy of the analytical way.

Thanks to versatility of the analytical way to also consider BS activity as recommend by ITU and eventually present preselectors FAU and 450connect suggest to replace the current 3-frequency approximation algorithm by the analytical way.

# References

[1] SE7(18)102 Reference Values for RX intermodulation, source: FAU

[2] SE7(18)028, Intermodulation issues, source: Motorola

[3] SE7(18)103, Status of SEAMCAT IM plugin validation, source FAU & 450connect

[4] Draft of ECC Report 283, Compatibility and sharing studies related to the introduction of broadband and narrowband systems in the bands 410-430 MHz and 450-470 MHz

[5] Oppenheim, Alan V., and Ronald W. Schafer. Discrete-time signal processing. Pearson Education, 2014

[6] H. V. Sorensen; C. S. Burrus; D. L. Jones, A new efficient algorithm for computing a few DFT points, IEEE International Symposium on Circuits and Systems, Year: 1988

[7] Report ITU-R M.2292-0, Characteristics of terrestrial IMT-Advanced systems for frequency sharing/interference analyses, 12/2013

[8] SE7(18)106 Impact of LTE BS Activity Factor on TETRA RX Intermodulation, source: FAU &450connect